ISSN 2229-5518

Application of Control Theory in the Efficient and Sustainable Forest Management

Md. Haider Ali Biswas, Munnujahan Ara, Md. Nazmul Haque, Md. Ashikur Rahman

Abstract— This paper focuses on the necessity of forest management using the model of control theory. Recent researches in mathematical biology as well as in life sciences closely depend on control theory. Various popular research papers have been received considerable attentions by engineers and research scholars due to the fact that it has been the central and challenging area of research for its wide range applications in the diverse fields. In this study we have briefly mentioned some of the fields in which these challenges are present, specially sustainable forest management is one of the warming issues in the present cencury. Our main objective in this paper is to investigate the scopes and applications of control theory in real life situation, specially the applications of control theory in the efficient and sustainable forest growth. A particular case of Sundarbans, the largest mangrove forest in the world is discussed with illustrative examples.

Index Terms— Control theory, Malthusian model, Logistic equation, Sundarban mangrove.

---

1 INTRODUCTION

NOW-a-days the applications of Optimal Control (OC) theory have been increased surprisingly in almost all branches of modern science and engineering, specially the OC is playing significant, in some cases the dominant roles in the fields of aerospace engineering, medicine, agriculture and economics. See for examples [9], [14]. The theory of optimization continues to be an area of active research not only for the mathematicians but also for the engineers as an indication both of the inherent beauty of the subject and of its relevance to modern developments in science, industry and commerce. The optimal fuel landings of the space vehicle, the optimal strategies for the drug doses in the chemotherapy of infectious diseases along with all others are the fascinating challenges at present in the ongoing development of science and technology [28]. All the achievements in the field of optimal control theory are mostly due to Pontryagin. Although several authors made tremendous contributions to the further developments in this area, but *Pontryagin Maximum Principle* is still a milestone in OC theory. See for examples [6], [10], [16], [30] and references therein for details study on optimal control theory. However, we are mainly concerned in this paper to discuss the application of OC theory in the sustainable forest management. Many researches have been carried out over the years (see for examples [12], [17] and [37] for the recent development) emphasizing on the optimal strategies for an efficient and sustainable forest management due to the fact that forests are one of the best sources for saving lives of the human being from the poisonous greenhouse gases, from many natural disasters like cyclone and so on. European Tropical Forest Research Network (ETFRN) recently pointed out on the similar issues of forest management [20]. The people of the coastal region live depending on forests specially for a hunting, wood harvesting as well as collecting of honey (see for examples [23], [32]). Accordingly, in the economic point of view, forests help to enrich the national economy of a country. So steps should be taken so as to manage a sustainable and effective ecosystem where control theory is the essential tool. Different mathematical models were proposed for this purpose. Many studies have been undertaken to determine the optimal forest rotation length under different scenarios since the advent of the modern civilization. Some of these were focused on the optimal rotation length with the consideration of only timber value. Others searched for the optimal rotation length with the inclusion of both timber and nontimber benefits (See for examples, [27], [34], [36] and references within). These studies have provided important guidelines on how to manage the number of existing trees in the even-aged plantations. However, their applications in uneven-aged, or natural forests, are limited because age is no longer an appropriate variable under such circumstances. Also, in formulating a forest management plan / policy, particularly in the management of a large-scale (such as a regional or national scale) forest resource, it may be more relevant to determine how much timber should be harvested and what level of the forest stock should be maintained than to know when trees should be cut. However, in all the above mentioned models, the authors proposed the optimal managements of the forests only when the forests are full of trees. All those may bring a good result when the forests will be full of trees. In that case, the Malthusian model can give an optimal growth of trees after a certain time. In this study, we have proposed an alternative model discussing the Malthusian model and Logistic equation with illustrative example for the optimal controlling of the forest growth. A case study for survival of the existence of the largest mangrove forest in the world will be discussed for the sustainable management.

2 THE MALTHUSIAN MODEL

The Malthusian law of growth enunciated by Thomas

ISSN 2229-5518

Robert Malthusian in his research documentary "*An essay on the principle of population*" and concluded that "Population when unchecked, increase, with a geometric ratio". See for details in [24]. Over the years, the Malthusian growth model is still the granddaddy of all population models, which is simply based on the famous exponential growth law. We begin this section with a simple derivation of the model.

Let t denote the time and $p(t)$ denote the number of individuals present at time t. In practice $p(t)$ is non negative integer. We assume that $p(t)$ is continuously differentiable.

The growth rate of a population is the rate at which population changes. If the population $p(t)$ at time t changes to $p(t+\delta t)$, the average per capita growth rate at time δt is

$$\frac{p(t+\delta t)-p(t)}{p(t)\delta(t)}$$

Taking limit $\delta t \to 0$, we get the instantaneous growth rate per capita at time δt as

$$\lim_{\delta t\to 0}\frac{p(t+\delta t)-p(t)}{p(t)\delta(t)}=\frac{p'(t)}{p(t)}$$

Now let,

b =intrinsic birth rate.

= The average number of offspring born per individuals per unit time.

d = intrinsic death rate.

= The fraction of individuals of the population dies per unit time.

$r = b-d$

= intrinsic growth rate of the population.

= Excess of birth over death per unit time per individuals.

Now, we consider a single species of population, the growth model is then described by

$$\frac{p'(t)}{p(t)}=r$$

$$\Rightarrow p'(t)=rp(t) \qquad (1)$$

with the initial population

$$p(t_0)=p_0>0$$

We have the mathematical model described the growth of single spaces population as

$$p'(t)=rp(t),\ p(t_0)=p_0>0 \qquad (2)$$

The solution of this equation is $p(t)=ce^{rt}$

For the initial condition $p(t_0)=ce^{rt_0} \quad \Rightarrow c=\frac{p_0}{e^{rt_0}}$

$$\therefore\ p(t)=p_0e^{-rt_0}e^{rt}$$

$$\Rightarrow p(t)=p_0e^{r(t-t_0)} \qquad (3)$$

which is known as exponential law of growth or Malthusian law of growth.

The behavior of the solutions depend upon the growth rate. Now we will discuss the solution for following several cases.

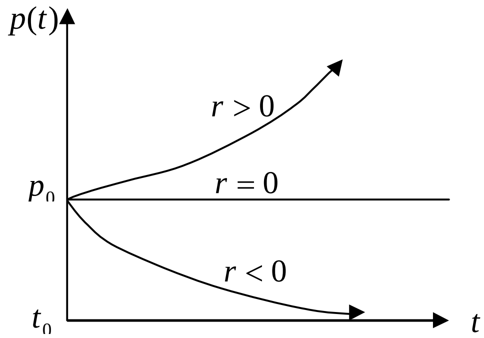

Fig. 1. Graph of the Malthusian model under 3 cases.

<u>Case 01:</u> For $r<0$, we have

$$p(t)=\lim_{t\to\infty}p(t)=\lim_{t\to\infty}p_0e^{r(t-t_0)}=0$$

This implies extinction of population. That is if the growth rate is negative, in the long run the population will be extinct.

<u>Case 02:</u> For $r=0$, we have

$$p(t)=\lim_{t\to\infty}p(t)=\lim_{t\to\infty}p_0e^{r(t-t_0)}$$

$$=\lim_{t\to\infty}p_0e^{0}=p_0$$

This refers constant population at zero growth rates.

<u>Case 03:</u> For $r>0$, we have

$$p(t)=\lim_{t\to\infty}p(t)=\lim_{t\to\infty}p_0e^{r(t-t_0)}=\infty$$

This is the case of unlimited growth.

So the Fig. 1 shows that for a positive growth rate, the trees of a forest after a certain time can be managed in its full naturally balanced equilibrum state.

3 The Logistic Equation

A typical application of the logistic equation (see [15] for details) is a common model of population growth, originally due to Pierre-François Verhulst in 1838, where the rate of reproduction is proportional to:

- The existing population
- The amount of available resources

Let $N(t)$ be the population of trees. The Malthusian model assumes a rate of growth proportional to the population $N'(t)=aN(t)$. This gives the exponential growth law $N(t)=e^{at}N(0)$, which is only accurate for relatively small values of $N(t)$; overcrowding and competition for resources lower the rate of growth. A more realistic model assumes a steadily decreasing, eventually negative growth coefficient $a(N)$.

In Malthusian model we assume that the rate in which an organism will reproduce or die remain constant.

In case of many other populations, such as population which exhibits exponential growth for a limited period ultimately approaches to some steady state. The growth of any population in a restricted environment must eventually be limited because that individual member complete with each other for the limited living space, natural resources and food. This forces the growth rate to decline,

the model, $N'(t) = aN(t)$ may be modified to

$$N'(t) = Nr(N) \tag{4}$$

where $r(N)$ is some decreasing function of N satisfying $r'(N) < 0$ and the simplest function regarding $r(N)$, having this property, is $r(N) = a - bN$, $a, b > 0$. Thus the model with initial population $N(t_0) = N_0 > 0$

$$N'(t) = N(a - bN), \qquad N(t_0) = N_0$$
$$= (a - bN(t))N(t)$$

This is the well-known *logistic equation*. This model gives good results for bacteria populations, even for human population [15] but does not describe accurately phenomena such as forest growth because of the fact that, the inhibiting effects of new trees on the growth rate are negligible until these have reached a certain "adult" size. Thus, the rate should be a function not only of $N(t)$ but also of $N(t-h)$ for a suitable time delay $h > 0$, leading to the equation called the *delayed logistic equation,*

$$N'(t) = (a - bN(t-h))N(t) \tag{5}$$

Similar delay effects are observed in the influence of overcrowding in human populations, for an elementary exposition of logistic equations with and without delays equation (4) and (5) has two equilibrium solutions. One is $N(t) \equiv 0$, whereas the other is

$$N(t) \equiv N_e = \frac{a}{b} \tag{6}$$

Assume tree seeds are planted, and trees are logged with seeding and logging rates $u_0(t)$ and $u_1(t)$ respectively. Let k be time it takes a seed to become baby tree. Then the equation becomes

$$N'(t) = (a - bN(t-h))N(t) + cu_0(t-k) - u_1(t) \tag{7}$$

where the coefficient c for $(0 \le c \le 1)$ accounts for the function of seeds that actually result in a tree.

To start the equation we need to know the forest population in an interval of length h,

$$N(t) = N_0(t) \qquad (t - h \le t \le t_0) \tag{8}$$

Now in order to attain the equilibrium population N_e at a certain optimal time $t = \bar{t}$ we have the solution,

$$N(\bar{t}) = N_0(t) \tag{9}$$

and we say that the population is at equilibrium at $t = \bar{t}$ but not necessarily afterwards. If the population is to stay at equilibrium, the target condition must be

$$N(t) = N_e \qquad (\bar{t} - h \le t \le \bar{t}) \tag{10}$$

This guarantees that $N(t) = N_e$ for all $t \ge \bar{t}$ if $cu_0(t-k) - u_1(t) = 0$ for $t \ge \bar{t}$. Target conditions of the form (9) are called Euclidean; those of the form (10) are called functional. To see the reason for this name, consider for instance the space $C[-h, 0]$ of continuous functions defined in the interval $-h \le t \le 0$. Given a function $y(t)$, denoted by $y_t(\tau)$ the section of $y(\tau)$ defined by

$$y_t(\tau) = y(t + \tau), \quad (-h \le \tau \le 0) \tag{11}$$

Then the target condition (10) can be written as an ordinary target condition in the space $C[-h, 0]$;

$$N_{\bar{t}}(\tau) = N_e \tag{12}$$

where N_e denotes the constant function. An optimal net profit problem is instance, to maximize the functional

$$J(t, u_1, u_2) = \alpha \int_0^t u_1(\tau) d\tau - \beta \int_0^t u_0(\tau) d\tau$$

with $\alpha, \beta \ge 0$ at some fixed time; the first term represents the profit from logging and the second, the cost of seeding. Clearly, $u_0(t), u_1(t)$ are nonnegative and it is reasonable to include upper bounds on both rates;

$$0 \le u_0(t) \le R, \ \ 0 \le u_1(t) \le S \tag{13}$$

Straight maximization of the profit may result in destruction of the forest at time $\bar{t}$, thus we supplement the problem with a (functional) target condition, say

$$|N(t) - N_e| \le \varepsilon, \ \ (\bar{t} - h \le t \le \bar{t}) \tag{14}$$

N_e the equilibrium solution (6), and the terminate seeding at time $\bar{t} - k$. If the equilibrium position is stable, this means the forest population will stay near equilibrium after $\bar{t}$. Admissible controls for this problem are pairs $(u_0(t), u_1(t))$, where u_0 is defined in $-k \le t \le \bar{t}$ and u_1 is defined in $0 \le t \le \bar{t}$, and satisfying (13) in their respective intervals of definition.

The logistic equation discussed above is in dimensional form which have some free parameters, but sometimes the dimensional form is transformed into nondimensional form in order to reduce the parameters which gives good results in the numerical treatments. We present here the nondimensional form of the logistic equation, $\frac{d\eta}{d\tau} = \eta(1-\eta)$, where $\tau = \frac{t}{t_*}$ and $\eta = \frac{N}{N_*}$

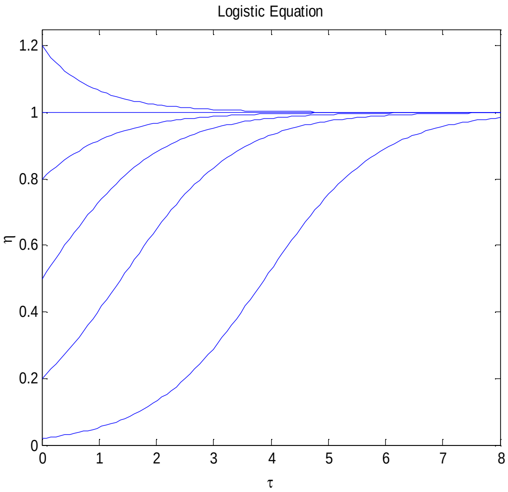

Fig. 2. The solution of nondimensional logistic equation

The numerical solution of the nondimensional logistic equation for the initial conditions 0.02, 0.2, 0.5, 0.8, 1 and 1.2 is shown in Fig. 2, where the lowest curve is the characteristic ´S-shape´ usually associated with the solution of the logistic equation. This sigmoid curve appears in many other models. See [4], [13] and [15] for details study on logistic equations and numerical applications.

Another growth model is described by the integrodifferential equation of the form,

$$N'(t) = \left(a - \left(\int_{-h}^{0} b(\tau) N(t+\tau) d\tau \right) \right) N(t) + cu_0(t-k) - u_1(t) \quad (15)$$

taking into account the inhibiting effects of new trees of all sizes on the growth rate. Our further research will be focused on that issue.

4 Application of ControlTheory

Numerous applications of control theory in the diverse fields have created this topic more challenging; specially an efficient and sustainable forest management is a crucial issue in the present world of climate change as well as the decay of ozone layer. In recent years, a number of economists and other experts have suggested sequestering carbon in forests to help mitigate the accumulation of greenhouse gases in the atmosphere. For more details studies see [1], [2], [21] and [22]. Forests currently store a substantial stock of carbon, amounting to 826 billion metric tons in trees and soil [11], and society can potentially remove carbon from the atmosphere by taking steps to increase this pool of carbon. These steps may include increasing the amount of carbon stored per hectare through management intensity or rotations ages (see for examples [21] and [37]) or increasing the area of land in forests. Carbon sequestration thus offers the promise of reducing the cost of greenhouse gas mitigation, which could lower the price of carbon and reduce global warming. Recent climate change is the global threat on the environment, where sustainable forest management is the burning issue worldwide. For details servey on greenhouse effects and climate change issues see [2], [7] and [34].

Forests also provide a large variety of services such as: timber production, recreation and landscape, natural habitat for numerous species, protections of watersheds, and protections of villages from avalanches and landslides and buffering as well. Thus from a socio-economic point of view, the optimal management of forests must take these multiple services into account. All the issues mentioned above indicate that a sustainable forest management is a crying need where the optimal policy strategy is the key.

5 The Sundarban: A case study

In this section, we will discuss the necessaity of forest management of a particular case of Sundarbans. The Sundarbans, situated both in Bangladesh and India is the world's largest delta, formed from the sediments brought down by three great rivers, the Ganges, the Brahmaputra and the Meghna which converge on the Bengal Basin. The forest consists of about 200 islands, separated by some 400 interconnected tidal rivers, creeks and canals. At 10,000 sq. km, it forms the largest mangrove forest in the world and it is the only mangrove tiger land on the earth. In 1947 the whole Sundarban mangroves were divided between India and Bangladesh (formerly East Pakistan), sharing 40% in India and 60% in Bangladesh. Although the two parts of the Sundarbans differ considerably in the nature and extent of investigations, conservations and managements due to belonging to the separate independent countries, the natural resources and beauties attract the global attentions simultaneously. As a results the Indian part of Sundarbans had been declared as a world heritage site in 1987 as *Sundarbans National Park* and Bangladesh part was declared as a world heritage site as *The Sundarbans* in 1997 by International Union for Conservation of Nature of UNESCO. The whole Sundarbans area was declared as Biosphere Reserve in 1989. See [5], [8], [25] and [32] for the details study about the history of Sundarbans and its local and global importance. However we are mainly concern to discuss the management about the Bangladesh part of Sundarbans. Presently the Sundarbans is one of the world's *new 7 wonders of nature* competitors' which will be declared at the end of 2011. The Sundarbans represents nearly half of the remaining forests of Bangladesh and is dominated by halophytic tree species such as *sundori* (*Heritiera fomes*) (from which Sundarbans derives its name), *gewa* (*Excoecaria agallocha*), *goran* (*Ceriops decandra*), *baen* (*Avicennia officinalis*), and *keora* (*Sonneratia apetala*). Sundarban is the habitat of many rare and endangered animals (*Batagur baska, Pelochelys bibroni*, *Chelonia mydas*), specially the Royal Bengal Tiger (*Panthera tigris*) and spotted deers. Javan rhino, wild buffalo, hog deer, and barking deer are now extinct from the area. The Sundarbans forest is the home to more than 400 tigers. However, the Royal Bengal Tiger is the king of all animals in the Sundarbans which have developed a unique characteristic of swimming in the saline waters, and are world famous for their man-eating tendencies. Apart from the Royal Bengal Tiger; *Fishing Cats*, *Macaques*, *Wild Boar*, *Common Grey Mongoose*, *Fox*, *Jungle Cat*, *Flying Fox*, *Pangolin*, *Chital*, are also found in abundance in the Sundarbans. However, widespread hunting and forest depletion has reduced the tiger's range and numbers in Sundarbans. Natural disasters as well as climate change are also affecting this world heritage. In 2007, Cyclone CIDR hit directly on the coastal areas of Sundarbans and almost destroyed the whole forests and thus destroyed the conservation of the wild animals. However, proper management of the Sundarbans is crucial for the sustainable dwelling of wildlife and their prey. The major steps against the unplanned or illegal wood-cutting as well as for increasing the number of trees in the forest should be taken. According to the *Rio Convention* proposed by the United Nations Conference on Environment and Development (UNCED) Bangladesh has the obligation to accomplish the required of the convention which is 'timely, reliable and accurate information on forests and forest ecosystems is essential for public understanding'. In the meantime, to ensure the protection for wildlife habitat and the management of natural resources, three areas within the forest have been designated as Wildlife Sanctuaries: Sundarbans West (715 sq. km), Sundarbans South (370 sq. km), and Sundarbans East (312 sq. km) by Bangladesh Government. A long term plan named *Bangladesh Tiger Action Plan 2009-2017* has been taken by Bangladesh Government for the conservation of tigers and other wildlife recourses as a part of forest management. See for details [3], [5].

5.1 Importance of the Sundarbans

The Sundarbans ecosystem is unique in many respects. The area experiences a subtropical monsoon climate with the annual rainfall of about 1600-1800 mm and several cyclonic storms. The biodiversity includes about 350 species of vascular plants, 250 fishes and 300 birds, besides numerous species of phytoplankton, fungi, bacteria, zooplankton, benthic invertebrates, molluscs, reptiles, amphibians and mammals. Species composition and community structure vary east to west, and along the hydrological and salinity gradients. Large areas of the Sundarbans mangroves have been converted into paddy fields over the past two centuries, and more recently into shrimp farms. The Sundarbans has been extensively exploited for timber, fish, prawns and fodder.

The Sundarbans is the only largest mangrove forest in the world managed for commercial timber production and has had a history of scientific management since 1879. In Bangladesh it is now managed by the Sundarban West Forest Division and Sundarban East Forest Division of the Forest Department, divided into 20 sections each harvested in turn on a 20-year cycle, with the three peripheral wildlife sanctuaries on the coast. Early management consisted of revenue collection by enforcing simple felling rules, subsequent enforcement of which reduced the amount of over-cutting of the four main timber species. A wildlife conservation plan prepared under the joint sponsorship of the World Wildlife Fund and the U.S. National Zoological Park emphasised management of the tiger and other wildlife as an integral part of sustainable forest and coastal management for both timber and the needs of the local population.

Approximately 2.5 million people lived in small villages surrounding the Sundarbans in 1981 which by 1991 had increased to 3 million. At nomination, some 35,330 people worked in the forest, 4,580 of whom collected timber and firewood, 1,350 collected honey and beeswax and 4,500 harvested the natural resources and hunted mainly deer, and 24,900 were fisherman and shrimp farmers. Today, the area provides a livelihood at some seasons of the year for an estimated 300,000 people. Some 4,500 people in Bangladesh are employed by contractors in the commercial logging of sundori and other timber, which is 45% of all that produced in state-owned forests. As well as construction timber they supply local newsprint paper, match and board mills (see for examples [8], [32]).

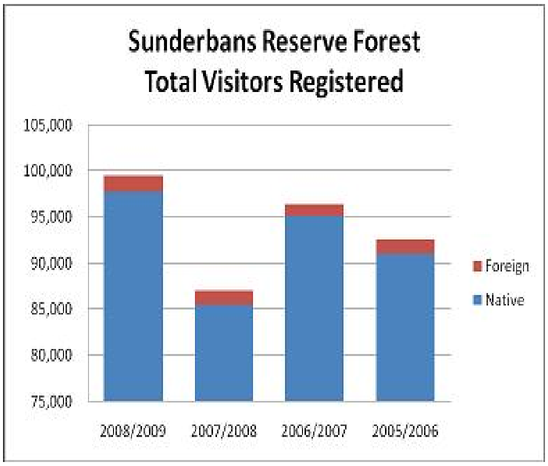

Fig. 3. The figure shows the statistics of total visitors visited the Sundarbans during 2005 to 2009 . Source: IPAC, 2009.

Local people are also dependent on the forests and waterways for firewood, charcoal, timber for boats and furniture, poles for house-posts and rafters, nypa palm thatch for roofing, grass for matting reeds for fencing, shells and reptile skins, with deer, fish, crabs and shrimps taken for food. The season for collecting honey and wax is limited to ten weeks from April 1st. Thousands of people, with permits from the Forest Department, enter the forest for nests. Before Cyclone Sidr, which has destroyed the fishing industry, more than 10,000 fishermen from as far away as Chittagong camped along the coast for 3-4 months in winter before returning home at the start of the monsoon season in April, and as many or more local people fished year-round [32]. In 1986 the average annual catch was 2,500 tonnes.

The important sector based on Sundarbans is *Tourism Industry*. The Sundarbans may be more attractive to the visitors and tourists due to its natural attractive beauties as well as the rare wild animals which will play a significant role both in the national and global economy. In 1996, about 500 foreign tourists plus 5,000 local tourists visited the area, most in the South Wildlife Sanctuary and in recent years most nearly 100,000 local and about 1,500 foreign tourists per year visited the Sundarbans. Although there is no potential for mass tourism but limited eco-tourism from October to April or May is possible. Recent research (see the Fig. 3) on Sundarbans showed that the number of national and international visitors and tourists are increasing.The figure reflects some variability in visitor numbers over the last five years, with the highest numbers in the year 2008-2009. This indicates that efficient and sustainable management of Sundarbans and availability of logistic facilities are necessary for enriching the tourism industry with strong revenue earning.

6 SUNDARBAN FOREST MANAGEMENTS

The future of the Sundarbans will depend upon the sustainable management of freshwater resources as much as on the conservation of its biological resources. Considerable researches both in nationally and internationally have been carried out on the Sundarbans ecosystem and its wildlife (see for examples [18], [25], [26] and [33]). Since the last few decades several national (see [29]) and international (see [23], [32]) leading NGOs carried out different (short-terms and long-terms) research projects on Sundarbans emphasizing on the sustainable managements to meet the future challenges for the next generations. They also emphasized on Collaborative forest management (CFM) which is loosely defined as a working partnership between the key stakeholders in the management of a given forest—key stakeholders being local forest users and state forest departments, as well as parties such as local governments, civic groups and nongovernmental organisations, and the private sector [12].

The world heritage Sundarban is the source of natural beauty to the tourists as well as to the environmentalists, where the *Royal Bengal* tiger is one of the attractions to the visitors. However, the natural disasters like flood, cyclone etc. are very common issues taking place in Bangladesh specially in that region. All most every year the natural disasters hit on this forest and as a result, the forest loses its trees and animals and consequently its beauty. Sometimes the unplanned cutting of the trees makes the forest to be destroyed day by day. So it becomes necessary to protect the Mangrove from the unavoided situations by making an optimal design for sustainable management. The above mentioned model for example can be applied for such efficient growth of the trees in the Sundarban Mangrove area. A successful application of the Malthusian model for the sustainable management of Sundarban Mangrove may ensure the long-term survival with its full beauty within a certain time applied. For a better understanding of the model we present here a simple example.

6.1 Example

Suppose the population of forest of Sundarban $p(t)$ given by the exponential growth law. Growth rate r is positive. How much time it will take to increase p_0 to double, where p_0 is the initial trees of forest.

Solution: Let the required time is T .
The exponential law of growth is

$$p(t) = p_0 e^{r(t-t_0)} \qquad (16)$$

For the given condition,

$$p(t_0 + T) = 2p(t_0)$$

$$\Rightarrow 2 = \frac{p(t_0+T)}{p(t_0)} \Rightarrow \frac{p_0 e^{r(t_0+T-t_0)}}{p_0 e^{r(t_0-t_0)}} = 2$$ [with the help of (16)]

$$\Rightarrow e^{rT} = 2 \Rightarrow rT = \log_e 2 \Rightarrow T = \frac{\log_e 2}{r}$$

After $T = \frac{\log_e 2}{r}$ times the trees in the forest will be double in compared to its initial state of the trees of the forest. This solution is shown in the Fig. 4 considering the growth rate r = 100.

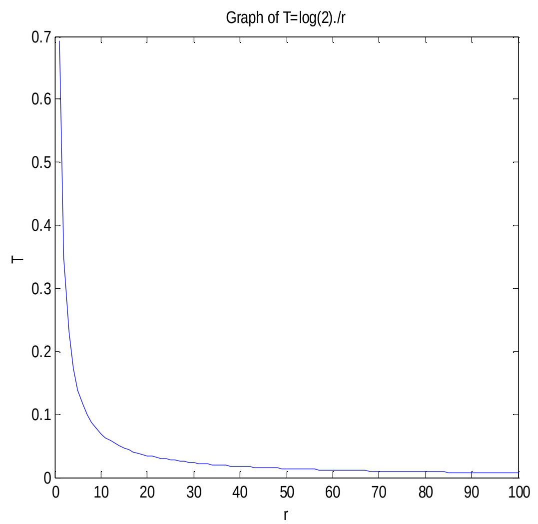

Fig. 4. The exponential growth of trees in the forest.

7 CONCLUSION

In Sustainable forest management is the warming issue due to the climate change and increasing the greenhouse gases. Optimal control strategy is the key to such conservation of forests. Among the different models for this optimal strategy Malthusian growth model is one which gives a good result where the logistic equation plays the vital role. This study investigates the application of control theory in the light of Malthusian model, which is used in controlling the forest growth and sustainable management. An illustrative example is presented taking into account the special case of the largest mangrove forest Sundarban. We claim that this study will play pioneer role in exploring further study in the field of control theory in agricultural and life sciences. The proposed model of controlling the optimal growth of the forests in the largest mangrove in world can be applied rigorously which may help to increase the different trees in the forests destroyed by the natural disaster every year in the coastal region and thus can make a natural equilibrium. we believe many more interesting conclusions will be brought under the different assumptions by using the optimal control strategy, and we hope to focus on such management issues in our future research, also we welcome more researchers to take part in the field.

ACKNOWLEDGMENT

The authors greatly acknowledge the logistic support provided by Mathematics Discipline, Khulna University, Bangladesh. We would also like to thank the reviewers for numerous helpful discussions and constructive suggestions for the modification of this paper.

REFERENCES

[1] Adams, R. M., D. M. Adams, J. M. Callaway, C. C. Chang and B. A. McCarl. 1993. Sequestering Carbon on Agricultural Land: Social Cost and Impacts on Timber Markets. *Contemporary Policy Issues*. 11: 76-87.

[2] Adams, D.M.; Alig, R.J.; McCarl, B.A.; Callaway, J.M.; and Winnett, S. M. 1999. Minimum cost strategies for sequestering carbon in forests. *Land Economics.* 75(3) 360-374.

[3] Ahmad, I. U., Greenwood, C. J., Barlow, A. C. D., Islam, M. A., Hossain, A. N. M., Khan, M. M. H. and Smith, J. L. D. 2009. *Bangladesh Tiger Action Plan 2009-2017*. Bangladesh Forest Department, Ministry of Environment and Forests, Government of the People's Republic of Bangladesh, Dhaka, Bangladesh.

[4] Allman, E. S. and Rhodes, J. A. 2003. Mathematical Models in Biology An Introduction. Cambridge University Press, New York.

[5] Altrell, D., Saket, M., Lyckebäck, L. and Piazza, M. *National Forest and Tree Resources Assessment 2005-2007, Bangladesh*. Implemented by Bangladesh Forest Department, Ministry of Environment and Forest (MoEF), Bangladesh Space Research and Remote Sensing Organization, Ministry of Defence, Bangladesh.

[6] Athans, M. and Falb, P. L. 1966. *Optimal Control*, McGraw-hill, New York.

[7] Backéus, S., Eriksson, L. O. and Garcia, F. 2005. Impact of climate change uncertainty on optimal forest management policies at stand level. Zerger, A. and Argent, R.M. (eds) *Proceeding on MODSIM 2005 International Congress on Modelling and Simulation.* Modelling and Simulation Society of Australia and New Zealand, December 2005.

[8] Bangladesh: Sundarbans Biodiversity Conservation Project, *ADB Validation Report*. Project Number: 30032, Reference Number: PCV: BAN 2008-37, Asian Development Bank, 2008.

[9] Biswas, M. H. A.; Huda, M. A.; Ara, M. and Rhaman, M. A. 2011. Optimal Control Theory and It's Applications in Aerospace Engineering. To be appeared in the *International Journal of Academic Research*, Vol. 3, No. 2, March 2011.

[10] Boltyyanskii, V. G. 1971. *Mathematical Methods of Optimal Control*. Holt, Rinehart and Winston, New York.

[11] Brown, S. 1998. Present and future role of forests in global climate change. In B. Goapl, P. S. Pathak, and K. G. Saxena (eds.), *Ecology Today: An Anthology of Contemporary Ecological Research*, International Scientific Publications, New Delhi, pp. 59-74.

[12] Carter, J. and Gronow, J. 2005. *Recent Experience in Collaborative Forest Management: A Review Paper.* CIFOR Occasional Paper No. 43. Center for International Forestry Research, Jakarta 10065, Indonesia.

[13] Chasnov, J. R. 2009. Mathematical Biology, Lecture Notes. Department of Mathematics, The Hong Kong University of Science and Technology, Hong Kong.

[14] Fattorini, H. O. 1999. *Infinite Dimensional Optimization and Control Theory*. Cambridge University Press, London.

[15] Gershenfeld, N. 1999. *The Nature of Mathematical Modeling*, Cambridge University Press, ISBN 978-0521-570954, Cambridge, UK.

[16] Glowinski, R. 1984. *Numerical Methods for Nonlinear Variational Problems* (2nd Edition), Springer-Verlag Publications, New York.

[17] Goetz, R. and Xabadia, A. 2003. Applications of Distributed Optimal Control in Economics – The Case of Forest Management. Working Paper, Economy Series E2003/37, Site A: Foundation Centre of Andalusian Studies, University of Girona, Spain.

[18] Gopal, B. and Chauhan, M. 2006. Biodiversity and its conservation in the Sundarban Mangrove Ecosystem. *Aquat. Sci. 68, 338–354.*

[19] Hoen, H. F. and Solberg, B. 1994. Potential and economic efficiency of carbon sequestration in forest biomass through silvicultural management. *Forest Science,* 40(3): 429-451.

[20] Holopainen, Jani and Marieke Wit (eds). (2008). *Financing Sustainable Forest Management.* Issue No. 49. European Tropical Forest Research Network (ETFRN), Tropenbos International, Wageningen, the Netherlands.

[21] Intergovernmental Panel on Climate Change (IPCC). 1996. Scientific-Technical Analysis of Impacts, Adaptations, and Mitigation of Climate Change. Report of Working Group II. Climate Change 1995: IPCC Second Assessment Report. Cambridge: Cambridge University Press.

[22] Intergovernmental Panel on Climate Change (IPCC). 2000. IPCC Special Report: Land Use, Land-Use Change, and Forestry. Cambridge: Cambridge University Press.

[23] Integrated Protected Area Co-Management (IPAC): Strengths, Weaknesses Opportunities, & Threats (SWOT) of Tourism in the Sundarbans Reserve Forest. USAID/Bangladesh, 2009.

[24] Malthus, T. J. 1798. *An Essay on the Principle of Population*. Chapter 1, p. 13 in Oxford World's Classics reprint.

[25] Manna, S., Chaudhuri, K., Bhattacharyya, S. and Bhattacharyya, M. 2010. Dynamics of Sundarban estuarine ecosystem: eutrophication induced threat to mangroves. *Saline Systems* 2010, 6:8.

[26] Musa, K. B. 2008. *Identifying Land Use Changes and It's Socio-economic Impacts; A Case Study of Chakoria Sundarban in Bangladesh*. Master's Thesis, Department of Computer and Information Science (IDA), Linköping University, Linkoping, Sweden.

[27] Noack, F. 2008.*The bioeconomics of livestock rearing and timber production in the Caspian forest*. Diploma Thesis in the study program of Landscape Ecology and Nature Conservation, Institute of Botany and Landscape Ecology, Ernst Moritz Arndt University Greifswald.

[28] Pinch, E. R. 1993. *Optimal Control and the Calculus of Variations*, Oxford University Press Inc., New York.

[29] Policy Brief on "Environmental Policy", *CPD Task Force Report*. 2001. Centre for Policy Dialogue, Dhaka, Bangladesh.

[30] Pontryyagin, L.; Boltyyanskii, L. S.; Gamkrelidze, R. V. and Mishchenko, E. F. 1964. *The mathematical theory of optimal processes*. Pergamon press, Oxford.

[31] Stavins, R. 1999. The Costs of Carbon Sequestration: A Revealed Preference Approach. *American Economic Review*. 89(4): 994 - 1009.

[32] Sundarbans National Park, India and The Sundarbans, Bangladesh, 2008. "Natural site datasheet from WCMC". World Conservation Monitoring Centre. http://www.unep-wcmc.org/sites/wh/pdf/Sundarbans.

[33] Tuhin, G., Gupinath, B. and Sugata, H. 2003. Application of a 'bio-engineering'technique to protect Ghoramara Island (Bay of Bengal) from severe erosion. *Journal of Coastal Conservation 9: 171-178.*

ISSN 2229-5518

[34] Van Kooten, G. Cornelius; Binkley, Clark S.; and Delcourt, Gregg. 1995. Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon services. *American Journal of Agricultural Economics*. 77(5): 365-374.
[35] Wilkie, M. L., Holmgren, P. and Castaneda, F. 2003. *Sustainable forest management and the ecosystem approach: two concepts, one goal*. Forest Management Working Papers, Working Paper FM 25. Forest Resources Development Services, Forest Resources Division, FAO, Rome, Italy.
[36] Xabadia, A. and Goetz, R. U. 2008. *The Optimal Selective Logging Regime and the Faustmann Formula*. Barcelona Economics Working Paper Series, Working Paper No. 353, University of Girona, Spain.
[37] Xiao, J., Kang, W., Yin, S. and Zhai, H. 2010. An Analytical Optimal Strategy of the Forest Asset Dynamic Management under Stochastic Timber Price and Growth: A Portfolio Approach. *Scientific Research: Low Carbon Economy*, 1, 25-28, 2010.